

On Algebraic Topology over Riemannian Manifolds.

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1 Abstract

In this note, a foundation of a framework is presented using Čech Obstructions in a topological space $\mathcal{X}$ over a bounded geometrical manifold M , for the purpose of geometrically bounded analysis.

2 Definitions, Corollaries and Theorems

Definition 2.1. Let a q -simplex of σ of $\mathcal{U}$ is an ordered collection of $q + 1$ sets chosen from $\mathcal{U}$, such that the intersection of all these sets is non-empty. This intersection is called the support of σ .

Definition 2.2. Definition 2.1 will be denoted as $|\sigma|$.

Corollary 2.3. Now let $\sigma = (U_i)_{i \in \{0, \dots, q\}}$ be such a q -simplex per Definition 2.1. The usage of a free abelian group A , per Definition 2.1 and Definition 2.2 spans over a bounded range of $\{0, \dots, p\}$ in a topological space $\mathcal{X}$.

Theorem 2.4. Thus per 2.3, 2.2, 2.1: Let $\mathcal{X}$ be a topological space equipped with an open cover $\mathcal{U} = \{U_i\}_{i \in I}$, and let M be a bounded Riemannian manifold. Let A be a free abelian group and $\check{C}^q(\mathcal{U}, A)$ the group of Čech q -cochains spanned over the q -simplices of $\mathcal{U}$ as in Definitions 2.2, and Corollary 2.3. Define the coboundary operator

$$\delta^q : \check{C}^q(\mathcal{U}, A) \rightarrow \check{C}^{q+1}(\mathcal{U}, A), \quad (\delta^q f)(\sigma) = \sum_{k=0}^{q+1} (-1)^k f(\sigma_{\hat{k}}),$$

where $\sigma_{\hat{k}}$ denotes the q -simplex obtained by omitting the k -th index. The Čech cohomology groups are then

$$\check{H}^q(\mathcal{U}, A) = \frac{\ker \delta^q}{\text{im } \delta^{q-1}}.$$

A geometric structure on M admits an extension across $\mathcal{X}$ if and only if its associated obstruction class $[\omega] \in \check{H}^q(\mathcal{U}, A)$ vanishes.