Methodology of Theorems and Proof-Writing.

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Contents

1 Synopsis:

THE previous chapter states the following as both a remark and theorem:
Let us impose remarks, and definitions per previous note.

Remark 1.1. Per the glossary's Definition 2.31 for $x = \frac{\ln(\pi^{\mathrm{e}})!}{\pi}$, $\forall x \in \mathbb{R}$. We have the following:

$$\frac{\ln(\pi^{\mathrm{e}})!}{\pi} = \frac{(\mathrm{e}\log(\pi))!}{\pi},\ x \neq 0. \tag{1}$$

Definition 1.2. Per Remark 1.1. Let us define the following $\forall x, n, p \in \mathbb{R}$.:

$$\mathcal{G}(x) = \int_n^p x \frac{\ln(\pi^{\mathrm{e}})!}{\pi} dx.$$

If and only If: $\mathcal{G}(x), x \neq 0$.

Theorem 1.3. *Per previous glossary. For the following integral that satisfies $\forall x \in \mathbb{R}$.:*

$$\int_e^\pi \frac{\ln(x)^e}{e} \cdot e^x dx \tag{2}$$

If and only If: $\forall x \in \mathbb{R}$, $\mathrm{e} \leq x \leq \pi$, $x \neq 0$, $\forall x, n, p \in \mathbb{R}$. Which then correlates to the following identity:

$$\int_n^p x \frac{\ln(\pi^{\mathrm{e}})!}{\pi} dx = \int_e^\pi \frac{\ln(x)^e}{e} \cdot e^x dx,\ x \in \mathbb{R}.$$

Which both converges starting from $[n, x]$, $n, p \neq 0$, $n, p \in \mathbb{R}$.
Thus If and only If: $\forall x \in \mathbb{R}$. Then the geometric figure admits the following hyperbola in $\mathbb{R}$.

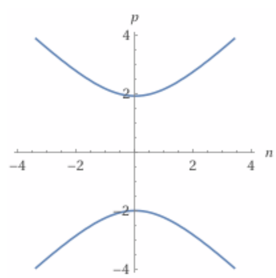

Figure 1. Geometric figure of the said hyperbola.

Definition 1.4. Per Definition 2.30. The DFT, for all $x, m, n \in \mathbb{R}$. And $X, N \in \mathbb{C}$.:

$$X_k = \sum_{m=0}^{N-1} x_m \mathrm{e}^{-i2\pi \frac{km}{N} n},\ k = 0, \ldots, N-1.$$

Per Dirichlet Series where $s \in \mathbb{C}$. And a_n is a complex sequence.:

$$\mathrm{D}(s) = \sum_{n=1}^{\infty} \frac{a_n}{n^s}.$$

Theorem 1.5. *Let the following for $\forall k, \in \mathbb{C}$, $\forall m \in \mathbb{R}$, $p \in \mathbb{N}$.:*

$$k = \prod_{m=1}^{p} \mathrm{e}^{-i2\pi \frac{km}{k}},\ k \in \mathbb{C}^+,$$

Then:

$$k = \prod_{m=1}^{p} \cos(-2\pi m) + i\sin(-2\pi m),\ k \in \mathbb{C}^{+}.$$

Therefore, by inference there is a following inequality in $\mathbb{C}^{+}$, $\forall n \in \mathbb{N}$, $n \neq 0$*.:*

$$\prod_{m=0}^{n} \cos(-2\pi m) + i\sin(-2\pi m) \leq n.$$

Thus for $\mathbb{C}^{+}$, $\forall n \in \mathbb{N}$, $n \neq 0$*.:*

$$\prod_{m=0}^{n} \cos(-2\pi m) + i\sin(-2\pi m)^{p} \leq n^{p},\ n \neq 0,\ p \neq 0,\ p \in \mathbb{Z}.$$

That admits a constant value x *in* $x \in \mathbb{R}$.

2 On Proof-Writing:

WRITING a proof involves multiple methods, constructive, chain inference, inference. All of whom shall be used per your theorem' case.
One caveat to add, is to know existing proofs and theorems that may help you bridge towards your proof. As how it happened with Fermat' Last Thoerem.
The following is a structural example of a proof by construction for a variable I in $\mathbb{C}^{+}$.

Proof. Per Theorem N:
(i) We will construct that for I assuming it is an irrational number in $\mathbb{C}$.
(ii) Therefore. Let $\forall I \in \mathbb{C}^{+}$. Observe that for $Z, Y, I, i\pi \in \mathbb{C}$.:

$$\mathcal{I} = (\mathcal{Z} + \mathcal{Y}) + i \cdot \pi,$$
$$\mathcal{I} - \mathcal{Z} = \mathcal{Y} + i \cdot \pi.$$

(iii) Per $i\pi$ being an irrational and transcendental number, per previous steps, summing $i\pi$ into $\mathcal{I}$ and for $p \neq 0$ would then prove by construction $\mathcal{I}$ to be an irrational number. ■

Every step of the proof shall be conditioned per (3), (2). As the importance of closing is really what makes a proof pass or break.

3 On Theorem-Writing:

WE would establish techniques, theorems, and definitions. Which will come as useful in our next chapter. The following is Rolle' Theorem. For all $c \in \mathbb{R}$.

Theorem 3.1. *A real function* $\mathcal{G}$ *is continuous on an interval* $[a, b]$*, that is properly closed and differentiable on the open interval* (a, b)*, and such that the identity* $\mathcal{G}(a) = \mathcal{G}(b)$ *holds.*
Then there exists at least one variable c in the open interval such that:

$$\mathcal{G}'(c) = 0,\ c \in \mathbb{R}. \tag{3}$$

Remark 3.2. We may notice that every definition is rigorously stated and defined. Because of the nature of Mathematics, one has to make sure that every statement communicates the theorem's logic accordingly. In other words, rigor and definitions for the purpose of our peers.

3.1 Expanding on Theorem-Writing:

GOING forward, the definition of a theorem should be in respect to earlier lecture and taking account of rigorous logic and proof-reading.
For example a theorem may be stated as follows:

Theorem 3.3. *Let $\mathcal{G}$ be a real-valued derivative of a function $\mathcal{F}$ on a closed interval $[a, b]$. If $\mathcal{G}$ is continuous on a closed interval $[a, b]$, then there exists at least one variable c in the open interval (a, b) such that:*

$$\mathcal{F}(b) - \mathcal{F}(a) = \mathcal{G}(c)(b - a),\ a, b, c \in \mathbb{R}. \tag{4}$$

Assuming the predicate below holds in $\mathbb{R}$.:

$$\mathcal{F}(b) - \mathcal{F}(a) \neq 0.$$

Remark 3.4. Such theorem are stated in respect to the MVT, and the burden of proof is on the reader to prove the theorem. Such reader may use earleir theorems, definitions, and a proof by construction to obtain the correct answer to the MVT.

It is also recommended that the reader should be able to prove one theorem by using another theorem, and so on. This is a good exercise to understand the nature of theorems and proof-writing.
It also reflects the real-world nature of the field, 'tinier' theorems leading to bigger theorems being proved by inference.

4 Final Words:

Introducing corollaries would be useful in the cases of complex proofs, where the focus is on working on torwards solving a bigger (proof) more elaborate problem, one example worth illustrating may be:

Corollary 4.1. *There exist X for Y in Z.*

Corollary 4.2. *There exist Y for Z that is non-zero.*

Proof. Proving per induction from corollaries 4.1, and 4.2, there exists an indentity P based on 3.5 and 3.6. ■

5 Going Forward:

FINISHING the note, we shall end by noting that more will come regarding proof-writing in Lean and Rocq. Lean (or Rocq) is a proof-assistant system that comes as useful to solve non-trivial mathematical theorems. Moreover, lecture of common litterature is highly recommended for such non-trivial tasks.

References

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