Lambert and Additional Plots.

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Contents

1 Synopsis: 2

2 Observations: 2

1 Synopsis:

THIS note will serve to observe several plots, and study their behavior, and cases.

2 Observations:

Definition 2.1. We define the following identity below in $\mathbb{C}$. Such that:

$$\ln(-1) = \ln(e^{i \cdot \pi}),\ i \in \mathbb{C}.$$

Thus:

$$\boxed{\ln(-1) = \ln(e^{i \cdot \pi}),\ i \in \mathbb{C}.}$$

Let us define the Lambert W function, named after Jean-Henri-Lambert, for $x \in \mathbb{C}$. Such that:

$$\exists W(z) \in \mathbb{C},$$
$$f(z) = z \cdot e^{z},\ f(z) = W(z).$$

Corollary 2.2. *Let us define the following relation for $z, z' \in \mathbb{C}$, which states:*

$$W(z') = W(z),\ z', z \in \mathbb{C}.$$

Proof. Per corollary 2.2, $\ln(-1) = \ln(e^{i \cdot \pi})$ can be expressed by construction in the form of:

$$W(\ln(-1)) = W(\ln(e^{i \cdot \pi})) \tag{1}$$

(i) By the following identity of $\ln(-1) = \ln(e^{i \cdot \pi})$ and $-1 = e^{i \cdot \pi}$.
(ii) And that the identity of $\ln(-1) = \pi \cdot i$, by construction yields the identity $\ln(-1) = \ln(e^{i \cdot \pi})$.
(iii) Thus proving by construction the following identity for $e^{i \cdot \pi}$. This completes the proof. ∎

Lemma 2.3. *Defining the following sequence $\mathcal{S}_k$ for $k \in \mathbb{N}$, we have:*

$$\mathcal{S}_k = \sqrt[p]{k} \tag{2}$$

And thus we may write the next sequence $\mathcal{S}_{k+1}$:

$$\mathcal{S}_{k+1} = \sqrt[p]{k+1} \tag{3}$$

Thus we have a difference variable $n \in \mathbb{N}$, such that:

$$\mathcal{S}_k - \mathcal{S}_{k+1} = \{n\} \tag{4}$$

For our discrete sequence $\mathcal{S}_k$ in $\mathbb{R}$.

References

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