An In-Between Proof Before Algebraic Concepts.

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Contents

1 Synopsis:

LET the following theorems, proof, and definitions:

1.1 Definition Of $f(x)$ in $\mathbb{R}$:

Definition 1.1. Let us define the following function $f(a, qk)$ in $\mathbb{R}$:

$$f(a, qk) \in \mathbb{R},\ f(a, qk) \to \infty.$$

Where there is variables $\forall a, qk \in \mathbb{R}$.

1.2 Several Theorems before Additional Theorems:

Theorem 1.2. *For all $x \in \mathbb{R}$ that for $\frac{\zeta(3)}{\pi}$ in $\mathbb{R}$.:*

$$x \coloneqq \frac{\zeta(3)}{\pi},$$
$$\exists \pi^{\frac{1}{12}} > x, \forall x \neq 0, \forall x \in \mathbb{R}.$$

Proof. Per Theorem 1.2. Let the following in $\mathbb{R}$ by induction.:
(i) By the property of $\pi^{\frac{1}{12}}$ being transcendental, by the previously stated inequality in $\mathbb{R}$.:

$$\exists \pi^{\frac{1}{12}} > x, \forall x \neq 0, \forall x \in \mathbb{R},\ x \coloneqq \frac{\zeta(3)}{\pi}.$$

(ii) This conpletes the proof. ■

Remark 1.3. Combining both formulas below:

$$\pi^{\frac{1}{12}} - \sum_{n=1}^{p} \frac{\zeta(3)}{\pi} + \frac{\pi^{\frac{13}{12}}}{\zeta(3)}.$$

One may explore the following equations, and identities in a domain $\mathbb{D}$.:

$$\pi^{\frac{1}{12}} - \sum_{n=1}^{p} \frac{\zeta(3)}{\pi} = \mathcal{H}_n,$$
$$\mathcal{P}_n = \frac{\pi^{\frac{13}{12}}}{\zeta(3)}.$$

Remark 1.4. Building upon previous results (1.2, 1.3):

$$\mathcal{P} = \frac{\pi^{\frac{13}{12}}}{\zeta(3)} = \mathcal{P}_\phi \tag{1}$$

We can also write the following generalization for $\mathcal{P}_k$ in $\mathbb{R}$:

$$\mathcal{P}_k = \frac{\pi^{\frac{13}{12}}}{\zeta(k)}.$$

Remark 1.5. Building upon Remark 1.4, there exists an inequality of:

$$\mathcal{P}_k > \mathcal{P}_\phi > \zeta(3) > \mathcal{P}_x.$$

Where $\mathcal{P}_k$ is a near approximation of $\zeta(3)$ in a Dirichlet series. For $p \neq 0, p \in \mathbb{N}$. Noting the expression $\frac{\zeta(3)}{\pi}$, we will write the following theorems:

Theorem 1.6. *There exist the following equation in* $\mathbb{R}$. *And* $\forall a, q, k \in \mathbb{R}$. *And* $p \in \mathbb{N}$.*:*

$$\sum_{k=1}^{p} \frac{f(a,qk)}{k} + \left(\pi^{\frac{1}{12}} - \frac{\zeta(3)}{\pi}\right) - \pi = 0,$$

Then moving the left-hand-side π *to a right-hand-side* $\pm\pi$.*:*

$$\sum_{k=1}^{p} \frac{f(a,qk)}{k} + \left(\pi^{\frac{1}{12}} - \frac{\zeta(3)}{\pi}\right) = \pm\pi.$$

And an alternate forms by dividing $\zeta(3)$ *to the right-hand-side or by also moving* $(\pi^{\frac{1}{12}} - \frac{1}{\pi})$ *to the right-hand-side.:*

$$\sum_{k=1}^{p} \frac{f(a,qk)}{k} + \left(\pi^{\frac{1}{12}} - \frac{1}{\pi}\right) = \pm\frac{\pi}{\zeta(3)},$$

$$\sum_{k=1}^{p} \frac{f(a,qk)}{k} = \pm\frac{\pi}{\zeta(3)} - \left(\pi^{\frac{1}{12}} - \frac{1}{\pi}\right).$$

Remark 1.7. For $\pm\pi \in \mathbb{R}$, the plus-minus sign was added as a caution measure against rigor.

Corollary 1.8. *Per Remark 1.3, we define* $\sum_{k=1}^{p} \frac{f(a,qk)}{k} = \pm\frac{\pi}{(\pi^{\frac{1}{12}} - \frac{\zeta(3)}{\pi})}$ *to be less than* π *in* $\mathbb{R}$. *Moving on to the next section, additional theorems will be stated.*

2 Additional Theorems and more explorations:

2.1 Expanding Definitions and Theorems:

On expanding in more domains and several thoughts:

Theorem 2.1. *For a set* $\mathcal{S} \subset \mathbb{D}$ *in an arbitrary domain* $\mathbb{D}$*:*
A set $\mathcal{S}$ *of* $\mathbb{D}$ *defined values, is defined to be used as a sequence of functions* F_n *in* $\mathbb{D}$.

$$\mathcal{S} = \{F_n(a,qk), \dots\} \subset \mathbb{D}.$$

Where $\mathcal{S}$ *is a set of functions* F_n *with a symmetric opposite of* F_n *in* $\mathcal{S}$.

Remark 2.2. And thus, per 2.1, the domain $\mathbb{D}$ shall not be equivalent to $\mathbb{C}$. In which there may be applications in computational theory and formal methods.

3 Some Topology and More:

A continuous topological group of parallel points may be defined as for domains $\mathbb{S} \wedge \mathbb{O}$:

$$\mathcal{G}(\mathbb{S}_n, \mathbb{O}_n). \tag{2}$$

Thus such groups may be used to describe parallel subgroups $\mathcal{B}_n$ of a group $\mathcal{G}$.

4 Conclusion:

OUR paper, per abstract, covered several theorems and proofs that we establish per 1.4, 1.3, 2.1, 1.2.

References

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