A Glossary of Theorems and Definitions in Number Theory and Analysis.

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Contents

1 Synopsis:

THIS note provides a reference for theorems, definitions, and equations for the purpose of being reusable. References are cited at the bottom of each of the note where credit is due.
One caveat: $y(x), \lambda(x), \rho(x)$ are defined generically in this note.
However a proof will have the concrete function attributed to them. Moreover, the note acts as an active research note with new definitions, and theorems added regularly.

2 Observations and Theorems:

PER the following observations, let the following theorems, definitions, and remarks, be defined in $\mathbb{R}$.
The following are defined:

Theorem 2.1. *Let the following in* $\mathbb{R}$.*:* $\forall S \in \mathbb{R}$, $Baud(MIN) < Freq(S) < Baud(MAX)$. *Where* $Baud(MIN)$ *is the minimal baud rate and* $Baud(MAX)$ *the maximal baud rate:*

$$Freq(S) = \frac{Hi(S)}{Lo(S)} Weight(x),$$
$$-\frac{Freq(S)}{Weight(S)} = \frac{Hi(S)}{Lo(S)}.$$

Per Hartley's law, the weight of S *over its frequency* F*, may be equal of the division between the highest signal and lowest signal of* S.

Remark 2.2. There exist degenerate cases per 2.1, per the weight S over F. Where S is greater than F.

Theorem 2.3. *Let for any variables of the expression* $\forall a, \mu, \lambda, b, c \in \mathbb{R}$ *define:*

$$\mathcal{V} = \int (\lambda + \mu) \cdot \sqrt{ax^2 + bx + c}\, dx.$$

We define two equations for $\forall \mathcal{V}'$, $d(\lambda + \mu) \in \mathbb{R}$.*:*

$$\mathcal{V}' = (\lambda + \mu) \cdot \sqrt{ax^2 + bx + c}x \cdot \int x \, d(\lambda + \mu) \sqrt{ax^2 + bx + c}.$$

And another form of the above equation for $\mathbb{R}$.*:*

$$\pm \frac{\mathcal{V}'}{(\lambda + \mu)} = \sqrt{ax^2 + bx + c}x \cdot \int x \, d(\lambda + \mu) \sqrt{ax^2 + bx + c}.$$

For a variable $\mathcal{V}$, $d(\lambda + \mu)$, $\mathcal{V}' \in \mathbb{R}$.

Remark 2.4. Per Theorem 2.3, λ shall be non-zero in $\mathbb{R}$.

Corollary 2.5. *Per Theorem 2.3, we have the following:*

$$\mathcal{V} = \int (\lambda + \mu) \sqrt{ax^2 + bx + c}\, dx.$$

One way to rewrite theorem 2.2, is $\forall a, \mu, \lambda, b, c \in \mathbb{R}$.*:*

$$\frac{\mathcal{V}}{(\lambda + \mu)} = \int \sqrt{ax^2 + bx + c}\, dx.$$

In order to obtain a ratio from the integral of roots.

Theorem 2.6. *Thus let the following per Theorem 2.3:*

$$\mathcal{O}' + (\lambda+\mu+x)^{n-2} = (\lambda+\mu+x)^{n-1} + (\lambda+\mu+x)^{n},$$
$$\frac{\mathcal{O}'}{(\lambda+\mu+x)^{n-2}} = \frac{(\lambda+\mu+x)^{n-1} + (\lambda+\mu+x)^{n}}{(\lambda+\mu+x)^{n-2}},\ \mathcal{O}' \neq 0.$$

Let $\forall\mathcal{O} \in \mathbb{R}$ be defined in $\mathbb{R}$.:

$$\mathcal{O} = \frac{d}{dx}\mathcal{O}',\ \forall\mathcal{O}, \mathcal{O}' \in \mathbb{R},\ \mathcal{O}' \neq 0.$$

Theorem 2.7. *Let the following for any $\forall m, n \in \mathbb{R}^+$.:*

$$\mu(n,p) \coloneqq \sum_{n}^{p} M(n,p),\ \mu(n,p) \in \mathbb{R}^+ \tag{1}$$

$\forall\mu(n,p)$ There exist such that the following is true to $\forall n, p \in \mathbb{R}^+$.:

$$\mu(n,p) = p! +\ (n+1p)! +\ (n+2p)! +\ \ \ldots\ \ + (n+xp)!. \tag{2}$$

Theorem 2.8. *Let a set of rational numbers $\forall x \in \mathbb{R}$, in $\forall\mu(n,p) \in \mathbb{R}$, be in the following inequality:*

$$\frac{2\pi}{\mu(m,p)} < x < \mu(n,p),\ x \neq 0.$$

Theorem 2.9. *Let for any real $\forall x \in \mathbb{R}$. If: $x \neq 0$. Then:*

$$\mathcal{E}(x) = \frac{\gamma(x)}{\delta(x)+\lambda(x)},\ \mathcal{E}(x) \in \mathbb{R},\ \gamma(x) \neq 0.$$

Therefore, based on the definition $\mathcal{E}(x)$ in $\mathbb{R}$.:

$$\mathcal{P} = \prod_{n=1}^{3} \rho(x)x^n + \int \mathcal{E}(x)\,dx,$$
$$\mathcal{P} = \prod_{n=1}^{3} \rho(x)x^n + \int \frac{\gamma(x)}{\delta(x)+\lambda(x)}\,dx,$$
$$\mathcal{P} = \int \frac{\gamma(x)}{\delta(x)+\lambda(x)}\,dx + x^n\rho(x)^3,\ \mathcal{P}(x) \neq 0.$$

Separately, let the following for $\forall x, n \in \mathbb{R}$:

$$\prod_{n=1}^{3} \rho(x)x^n + \int \frac{\gamma(x)}{\delta(x)+\lambda(x)}\,dx \in \mathbb{R}.$$

There exist a solution in $\forall s = \mathcal{P}' \in \mathbb{R}$, of the following:

$$\mathcal{P}' = \prod_{n=1}^{3} \rho(x)x^n + \int \frac{\gamma(x)}{\delta(x)+\lambda(x)}\,dx,\ \mathcal{P}' \neq 0.$$

Separately, let the following $\forall x, \mathcal{L} \in \mathbb{R}$.:

$$\mathcal{L} + \lambda(x)^2 = \frac{\gamma(x)}{\delta(x)},$$
$$\mathcal{L} + \lambda(x) = \frac{\gamma(x)}{\delta(x)\lambda(x)},$$
$$\mathcal{L} = \frac{\gamma(x)}{\delta(x)\cdot\lambda(x)} - \lambda(x),\ (\delta(x)\cdot\lambda(x)) \neq 0.$$

Where the value of $\forall \Delta y \in \mathbb{R}$ *equals:*

$$= \frac{\ln x}{\Delta y},$$
$$\Delta y = \ln x,$$

Separately, there exists for $\forall I, r, x, \Delta x, n \in \mathbb{R}$*:*

$$I \coloneqq -r + \Big(\frac{\ln x}{\Delta n} + \frac{\Delta n}{\ln x}\Big)^m,\ r \neq 0,\ r \in \mathbb{R}^+.$$

For all variables of $\forall r \in \mathbb{R}$*. Being equal to each other, e.g:* $x = n = \ln x = m$*. Then the following equation:*

$$r = \Big(\frac{\ln x}{\Delta n} + \frac{\Delta n}{\ln x}\Big)^m,\ r \neq 0,\ r \in \mathbb{R}+.$$

Admits a transcendental result, If and only If: $r \neq 0$, $r \in \mathbb{R}$.

Theorem 2.10. *Let the following plots both a real and imiginary part for both. If:* $x, n \in \mathbb{R}$*. Then:*

$$\frac{7}{\log n} + \frac{363 \log 7}{140} = \sum_{n=1}^{m} \frac{\ln x}{n} + \frac{n}{\ln x},\ m = 7,\ \log n \neq \log 1.$$

And thus by computing the products of $(\frac{\ln x}{n} + \frac{n}{\ln x})$*.:*

$$\frac{7}{\log n} + \frac{\log^7 7}{5040} = \prod_{n=1}^{m} \frac{\ln x}{n} + \frac{n}{\ln x},\ m = 7,\ \log n \neq \log 1.$$

Theorem 2.11. *Let us define the following equations for* $\forall n \in \mathbb{R}^+$*:*

$$\mathcal{Z} = \frac{\phi(n^s!)}{\tau(n^s!)} + \frac{\phi(n^s!)}{\lambda(n^s!)} + \phi(n^s!),\ \mathcal{Z} \in \mathbb{Z}.$$

That admits the following alternate form:

$$\begin{aligned}\mathcal{Z}' &= \frac{\phi(n^s!)}{\tau(n^s!)} + \frac{\phi(n^s!)}{\lambda(n^s!)} + \phi(n^s!),\\ &= \phi(n^s!)(\frac{1}{\tau(n^s!)} + \frac{1}{\lambda(n^s!)}) + \phi(n^s!),\\ &= \phi(n^s!)(\frac{1}{\tau(n^s!)} + \frac{1}{\lambda(n^s!)} + 1).\end{aligned}$$

Alternatively, we can assume another equation:

$$\phi(s, n) = \frac{(s-3)}{\tau(n^s!)} + \frac{(s-2)}{\lambda(n^s!)} + \frac{(s-1)}{f'(n^s!)} + s,\ s \neq 0.$$

Assuming $\phi(s, n) \neq 0$*. And* $\phi(s, n) \in \mathbb{R}$.

Definition 2.12. Let a theorem of the following equations for any $\forall C, x, n \in \mathbb{R}$:

$$\mathcal{C} = \int \frac{\lambda(y)\sqrt{x}}{x}\, dx = -\prod_{n=1}^{y} \rho(x) x^n + \mathcal{C}',$$
$$\mathcal{C} = \int \frac{\lambda(y)\sqrt{x}}{x}\, dx + \prod_{n=1}^{y} \rho(x) x^n = \mathcal{C}'$$

Admits a set of results in the domains of $\mathbb{R}^+$. Alternatively we can suppose the following equations $\pm\mathcal{E} \in \mathbb{R}$.

$$\gamma(x) + \lambda(x) + \prod_{n=2}^{4} \frac{1}{\rho(x)}^{n} = \pm\mathcal{E},$$
$$\gamma(x) + \lambda(x) + \frac{1}{\rho(x)} \frac{1}{\rho(x)}^{2} \frac{1}{\rho(x)}^{3} \frac{1}{\rho(x)}^{4} = \pm\mathcal{E},$$
$$\gamma(x) + \lambda(x) + \frac{1}{\rho(x)^{10}} = \pm\mathcal{E}.$$

If, assuming all variables are non-zero.

Definition 2.13. Let us define the following for $\forall \Delta x, x, y \in \mathbb{R}$:

$$\Delta x = y + \gamma(x),\ \gamma(x) < \Delta x.$$

Separately, we have the following for $x \in \mathbb{R}$:

$$\mathrm{A} = \gamma(x) + \Delta x + \frac{1}{\lambda(x)},\ \lambda(x) \neq 0.$$

Separately, we have the following for $\forall x, y \in \mathbb{R}$:

$$\frac{x}{\lambda(x)} + \gamma(x) = \frac{x}{y} + \mathrm{Y}',$$
$$\frac{x}{\lambda(x)} + \gamma(x) - \frac{x}{y} = \mathrm{Y}',\ \Delta x \in \mathbb{R},$$
$$\lambda(x) + \gamma(x) - \frac{x}{y} = \frac{\mathrm{Y}'}{x},\ x \neq 0.$$

For all valid numbers in the domains of '$\mathbb{R}$'.

Theorem 2.14. *Let an equation of the following for $\forall \mathcal{L}, x, y, z \in \mathbb{R}$:*

$$\mathcal{L} = \iiint (xyz)^3\, dx \wedge dy \wedge dz,$$
$$c_1yz + c_2z + c_3 + \frac{1}{64} + x^4 + y^4 + z^4 = \iiint (xyz)^3\, dx \wedge dy \wedge dz.$$

Admits the following equation $\forall c, x, y, z \in \mathbb{R}.$:

$$(c_1yz + c_2z + c_3 + \frac{1}{64} + x^4 + y^4 + z^4)^{n-2} + \sum_{n=1}^{p} \frac{(c_1yz + c_2z + c_3 + \frac{1}{64} + x^4 + y^4 + z^4)^{n}}{(c_1yz + c_2z + c_3 + \frac{1}{64} + x^4 + y^4 + z^4)^{n-1}} = \pm\mathcal{C},$$

Definition 2.15. We define Euler–Mascheroni constant:

$$\gamma = \lim_{n\to\infty} (\sum_{k=1}^{n} (\frac{1}{k} - \log(n))). \tag{3}$$

Which may be rewritten to per M. Ram Murty and N. Saradha:

$$\gamma(a, q) = \lim_{n\to\infty} \sum_{k=0}^{n} (\frac{1}{a + kq} - \frac{\log(a + nq)}{q}). \tag{4}$$

Definition 2.16. The following is the closed form of a series of low-integer-order polylogarithms:

$$\frac{\partial}{\partial z} \operatorname{Li}_n(z) = \frac{\operatorname{Li}_{n-1}(z)}{z}.$$

Definition 2.17. The following is a series of the exponential function:

$$\sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z.$$

Theorem 2.18. *The following is a series of Euler–Mascheroni constants over k:*

$$\sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)},\ qk \neq 0.$$

Definition 2.19. The following is a series of k over Euler–Mascheroni constants:

$$\sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k},\ k \neq 0.$$

Definition 2.20. The following inequality may also be defined:

$$\sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} < x < \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)},\ x \neq 0.$$

Definition 2.21. Applying the previous theorem laid out on previous papers. We satisfy the following:

$$\begin{aligned} y &= \pm z \frac{b}{c!},\ \pm z \in \mathbb{R}^+ \\ \frac{y}{\pm z} &= c!,\ \pm z \neq 0 \\ -c! &= \pm \frac{y}{\pm z}. \end{aligned}$$

For any domain $\mathbb{R}^+$. Where the following is a Taylor Series of:

$$z = \frac{x^n}{n!},\ n \neq 0. \tag{5}$$

Theorem 2.22. *Let for $\forall y, \pm z, a, q, k, x \in \mathbb{R}$:*

$$\begin{aligned} y &= \sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} < x < \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)}, \\ \frac{y}{\pm z} &= \frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} < x < \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z}. \end{aligned}$$

Now in order to continue further, we need to solve $\forall x \in \mathbb{R}$.:

$$\begin{aligned} \sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} &\leq x < \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)}, \\ \sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} &\leq x < \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)}, \\ \sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)} &\leq x, \end{aligned}$$

Therefore, for $x = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}, \forall x \in \mathbb{R}$.

$$\begin{aligned} \pm z + y &= \sum_{k=1}^{\infty} \frac{\gamma(a, qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a, qk)}, \\ y &= \frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z}. \end{aligned}$$

Then for: $x = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}, \forall x \in \mathbb{R}.$*:*

$$\pm z + \frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z} = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)},$$
$$\frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z} = \frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z}, \pm z \neq 0.$$

Definition 2.23. We define the Golden ratio constant ϕ as:

$$\phi = \frac{1+\sqrt{5}}{2}, \forall \phi \in \mathbb{R}.$$

And the Supergolden-ratio constant ψ as:

$$\psi = \frac{1 + \frac{29+3\sqrt{93}}{2} + \frac{29+3\sqrt{93}}{2}}{3}, \forall \psi \in \mathbb{R}.$$

And the Kepler-Bouwkamp constant K as:

$$\mathrm{K} = \prod_{n=3}^{\infty} \cos(\frac{\pi}{n}),$$
$$= \cos(\frac{\pi}{n-1})\cos(\frac{\pi}{n})\ldots, \forall K \in \mathbb{R}.$$

And. For an equation of the previous result per Theorem 2.19, we can define the following:

$$\mathrm{A} = \pm z + \frac{\sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)}}{\pm z},$$
$$\pm z^2 = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)},$$
$$\mathrm{A}' = \pm z^2 = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k} + \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)},$$
$$\mathrm{A}'' = \pm z^2 - \sum_{k=1}^{\infty} \frac{k}{\gamma(a,qk)} = \sum_{k=1}^{\infty} \frac{\gamma(a,qk)}{k}, \ k \neq 0, \ \forall \mathrm{A} \in \mathbb{R}.$$

Theorem 2.24. *Let for the following result for* $\forall x \in \mathbb{R}.$*:*

$$\sqrt{2} + 2 = \sqrt{2} + x,$$
$$2\sqrt{2} + 2 = x,$$
$$2(\sqrt{2} + 1) = x,$$

Then we can solve for $l = x, \forall l \in \mathbb{R}.$

$$l + r \leq n - 1,$$
$$2(\sqrt{2} + 1) + r \leq n - 1.$$

Also. $\forall x$ *There exist for* $r \in \mathbb{R}^+$.

$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} \leq \frac{n-1}{2} + n - 1,$$
$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} - \frac{n-1}{2} \leq n - 1.$$

Theorem 2.25. *Let a general theorem of the following for $\forall x \in \mathbb{R}$ be:*

$$\mathcal{B} = 2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} - \frac{n-1}{2},$$
$$= \frac{\frac{\frac{n-1}{2}-4}{2} - \frac{n-1}{2}}{2(\sqrt{2}+1)}.$$

Then. Substituting $\frac{n-1}{2} = \phi$, $\forall \phi \in \mathbb{R}$.:

$$\mathcal{C} = 2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} - \phi,$$
$$= \frac{\frac{\phi-4}{2} - \phi}{2(\sqrt{2}+1)},$$
$$\mathcal{C}' = \frac{\phi - 4}{2} - \phi.$$

Theorem 2.26. *Let the following constant $C \in \mathbb{R}$. admit a derivative of the following:*

$$\mathcal{C}' = \frac{\phi - 4}{2} - \phi,$$
$$\mathcal{C}' = \frac{\frac{1+\sqrt{5}}{2} - 4}{2} - \frac{1+\sqrt{5}}{2}.$$

Theorem 2.27. *Let the following equation for $\forall n \in \mathbb{R}$.*

$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} \leq \frac{n-1}{2} + n - 1,$$
$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} - \frac{n-1}{2} \leq n - 1,$$

That admits the following solution of $(2(\sqrt{2}+1) + \frac{\frac{n-1}{2}-4}{2}) \in \mathbb{R}$.:

$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} \leq 0.$$

Definition 2.28. Let the following equation for $\forall n \in \mathbb{R}$.:

$$2(\sqrt{2}+1) + \frac{\frac{n-1}{2} - 4}{2} = x,$$

Thus for the following variable $n = 5$, $\forall n \in \mathbb{N}^{+}$.:

$$2(\sqrt{2}+1) + \frac{5}{2} = x,$$
$$\sqrt{2} + 2 + \frac{5}{2} = x.$$

And the following for $x \in \mathbb{R}$.:

$$2 + \frac{5}{2} = \frac{x}{\sqrt{2}}.$$

Theorem 2.29. *Let the following derivative, and primitive function for $\forall x \in \mathbb{R}$.:*

$$f'(x) = \frac{\ln(x)^{\mathrm{e}}}{\mathrm{e}} \cdot \mathrm{e}^{x},$$
$$f(x) = \int_0^1 f'(x)\,dx,$$
$$= \int_0^1 \frac{\ln(x)^{\mathrm{e}}}{\mathrm{e}} \cdot \mathrm{e}^{x}\,dx.$$

Theorem 2.30. *Let the following equation for* $\forall x \in \mathbb{R}$.*:*

$$x = 2(\sqrt{2}+1) + \frac{5}{2}.$$

Thus for $\forall x$ *in* $\mathbb{R}$*, the following integral is defined as:*

$$\int_0^1 \frac{\ln(x)^{\mathrm{e}}}{\mathrm{e}} \cdot \mathrm{e}^x \, dx.$$

Then there exists the following definite integral admits a solution $\mathrm{R} \in \mathbb{R}$.

$$\begin{aligned} \mathrm{R} &= \int_0^1 \frac{\ln(x)^{\mathrm{e}}}{\mathrm{e}} \cdot \mathrm{e}^x \, dx, \\ &= \int_0^1 \frac{\ln(2(\sqrt{2}+1)+\frac{5}{2})^{\mathrm{e}}}{\mathrm{e}} \cdot \mathrm{e}^{2(\sqrt{2}+1)+\frac{5}{2}}, \\ &= \mathrm{e}^{7/2+2\sqrt{2}} \log^e(\frac{9}{2}+2+\sqrt{2}) + \mathcal{C}. \end{aligned}$$

Theorem 2.31. *Let the following definite integral for* $\forall x, y, \pm D \in \mathbb{R}$.*:*

$$\int_C^\pi \frac{\ln|x| \cdot f(y)f'(x)}{x^n y!} \, dx = \pm D, \ \pm D \neq 0.$$

With a definite integral that admits a solution of:

$$\int_C^\pi \frac{\ln|x| \cdot f(y)f'(x)}{x^n \cdot y!} \, dx = \big[\frac{\ln|x| \cdot f(y)f'(x)}{x^n \cdot y!}\big]_C^\pi + D.$$

Let us impose two inequalities for $\forall x, y, n \in \mathbb{R}$.*:*

$$\begin{aligned} \frac{\pi}{\frac{\ln|x| \cdot f(y)f'(x)}{x^n \cdot y!}} &< \frac{\ln|x| \cdot f(y)f'(x)}{x^n \cdot y!} < \sum_{n=1}^{\infty} \frac{\ln|x| \cdot f(y)f'(x)}{x^n \cdot y!}, \\ \frac{\pi}{\ln|x| \cdot f(y)f'(x)} &< \ln|x| \cdot f(y)f'(x) < \sum_{n=1}^{\infty} \ln|x| \cdot f(y)f'(x). \end{aligned}$$

Theorem 2.32. *Let the following equation for* $\forall x, y, n, C \in \mathbb{R}$.*:*

$$\pm \frac{\gamma(x-y)}{|x-y|^n}! = \pm \frac{\sum_{n=1}^p \gamma(x-y)}{\sum_{n=1}^p |x-y|^n} + C, \ x \neq y, \ y \neq 0, \ x \neq 0.$$

Definition 2.33. Let the transform for $N \neq 0$, $N \in \mathbb{N}$. Be the following:

$$X_k = \sum_{m=0}^{N-1} x_m \mathrm{e}^{-i2\pi \frac{km}{N} n}, \ k = 0, \ldots, N-1.$$

The inverse notation for $n \neq 0, N \neq 0$, $N \in \mathbb{N}$. Is given by:

$$x_k = \frac{1}{N} \sum_{m=0}^{N-1} X_k \mathrm{e}^{i2\pi \frac{km}{N} n}, \ k = 0, \ldots, N-1.$$

For $x_0, \ldots, x_{n-1}$ be complex numbers. Where $\mathrm{e}^{-i2\pi/n}$ is a primitive nth root of 1.

Theorem 2.34. *Let the following identitiy in* $\mathbb{R}$.*:*

$$\frac{\ln(\pi^{\mathrm{e}})!}{\pi} = (\mathrm{e}\log(\pi))!.$$

And for $\mathcal{H} \in \mathbb{R}$.:

$$\mathcal{H} \coloneqq \frac{\ln(\pi^{\mathrm{e}})!}{\pi} + \frac{(\mathrm{e}\log(\pi))!}{\pi},\ \forall \mathcal{H} \in \mathbb{R}.$$

From the following for $x = \frac{\ln(\pi^{\mathrm{e}})!}{\pi}$, $\forall x \in \mathbb{R}$.:

$$\mathcal{H} \coloneqq \frac{\ln(\pi^{\mathrm{e}})!}{\pi} + \frac{(\mathrm{e}\log(\pi))!}{\pi},\ \forall \mathcal{H} \in \mathbb{R}.$$

We then would establish another identity $\forall x \in \mathbb{R}$.:

$$x = \pm\frac{(\log(\pi)!)(e\log(\pi))!}{\pi},\ x \neq 0.$$

Remark 2.35. The previous identity of Theorem 2.31 will be later used, bear it in mind.

Definition 2.36. Defining the Chvátal–Sankoff constants for $\gamma_k, n, k \in \mathbb{R}$.:

$$\gamma_k = \lim_{n\to\infty} \frac{\mathcal{E}\lambda_{n,k}}{n},\ n \neq 0. \tag{6}$$

3 Additional Remarks:

THE previous section, which establishes 31 definitions and theorems. Will now serve as a basis for plotting. Which in the next chapter 'On Theorems' will be used to plot some of our theorems. But before, let us impose the following remark in $\mathbb{R}$.:

Remark 3.1. Per Theorem 2.31, for $x = \frac{\ln(\pi^{\mathrm{e}})!}{\pi}$, $\forall x \in \mathbb{R}$. We have the following:

$$\frac{\ln(\pi^{\mathrm{e}})!}{\pi} = \frac{(\mathrm{e}\log(\pi))!}{\pi}, x \neq 0,\ \forall x \in \mathbb{R}. \tag{7}$$

Corollary 3.2. *Per Remark 3.1 for each value of x, there exists such identities:*

$$\frac{\ln(\pi^{\mathrm{e}})!}{\pi} = \frac{(\mathrm{e}\log(\pi))!}{\pi}, x \neq 0,\ \forall x \in \mathbb{R}.$$

Defined in $\mathbb{R}$. And for $x \in \mathbb{R}$.

Corollary 3.3. *For each index n in a numerical sequence $x, p, n \in \mathbb{R}$, it is defined as the following:*

$$x_n = \frac{p}{n+1},\ n \neq 0.$$

There exist by the property of parity if and only if $n + 1 = 2p + 1$, then the sequence contains odd indices.

Definition 3.4. Let us define a sequence of tensor products for $\forall \mathcal{X}_n, \mathcal{Y}_n \in \mathbb{R}$:

$$\mathcal{Z}_n = \sum_{w=1}^{+\infty} (\mathcal{X}_{w+n} \otimes \mathcal{Y}_{w+n}).$$

Where $\mathcal{Z}$ is a tensor product of the sequences $\mathcal{X}_n$ and $\mathcal{Y}_n$ from a Riemannian plane.

Definition 3.5. Let us define a double integral for $\forall x, y \in \mathbb{R}$ with a function $f(x, y)$ over a domain D in a xy-plane (who we will refer to as a squared-plane of values x and y):

$$\mathcal{I}_n = \iint_{\mathcal{D}} f(x,y) \otimes \mathcal{Z}_n\, dx \wedge dy,\ x, y,\ f(x,y) \in \mathbb{R}.$$

Where $\mathcal{I}_n$ is a double integral of the function $f(x, y)$ over the domain D in the xy-plane.

4 Going Forward:

THE ten pages of theorems now comes to an end, stating them here was the first step to then develop a framework later in the series of papers. We will next plot and analyze our results based of our theorems and definitions. We will now proceed to plot and analyze the following theorems to analyze their convergence and general behavior.

References

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