Construction Notes on the Cauchy Momentum Equation.

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September 23, 2026

1 Abstract

We present a construction of the Cauchy momentum equation in normalized form. Beginning from the vector form of the equation, we derive an equivalent canonical representation by eliminating the mass density factor, and prove the equivalence of both forms. The resulting expression provides a minimal, self-contained foundation for subsequent analytical or computational treatment of momentum transport in continua.
Keywords: PDE, Analysis, Continuum Mechanics.

2 Canonical Form of the Cauchy Momentum Equation

Proof. (i) The Vector Form of the Cauchy Equation, where ρ is the mass density:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}. \tag{1}$$

(ii) Transformed when dividing ρ on both sides from Equation 1:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} + \frac{1}{\rho} \mathbf{f}. \tag{2}$$

(iii) Applying it back from Equation 1 to Equation 2, per Equation 1 and Equation 2 we then have:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{\nabla \cdot \boldsymbol{\sigma} + \mathbf{f}}{\rho} \tag{3}$$

Thus Equation 1 and Equation 3 are equivalent forms of the Cauchy momentum equation. ■