Second Additional Write-up for Future Problems.

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Contents

1 Abstract:

2 Lemmas and Proofs:

THE following notes are stated in a reusable manner for future work in Abstract Algebra and Differential Geometry.

2.1 Lemmas and Theorems for Tensors:

Lemma 2.1. *There exist a following identity of factor p of an index of a tensor $\mathcal{I}_{kij}$ at i, j, k in $\mathbb{N}^+$.*

$$||\mathcal{E}_{ijk}||^p = (p \cdot \mathcal{I}_{kij}). \tag{1}$$

The following equation holds for $\forall \mathcal{E} \neq 0 \wedge \forall p \neq 0$.
For $i, j, k \subset \mathbb{N}^+$.

Remark 2.2. Per Lemma 2.1 There exist a specific nonlinear graph of $\mathcal{E}$, if and only if $p < 0$.

Definition 2.3. Per 2.1. Let the equation of $p, n \in \mathbb{N}^+$ be defined for two tensors $\mathcal{E}_{ijk} \wedge \mathcal{I}_{kij}$.

$$||\mathcal{E}_{ijk}||^{p+n} = (p \cdot \mathcal{I}_{kij})^n. \tag{2}$$

Remark 2.4. We could divide to p on both sides of the equation per 2.3:

$$\begin{aligned} ||\mathcal{E}_{ijk}||^{p+n} &= (p \cdot \mathcal{I}_{kij})^n, \\ \frac{||\mathcal{E}_{ijk}||^{p+n}}{p} &= \mathcal{I}^n_{kij},\ \forall \mathcal{E}_{ijk} \neq 0,\ \forall p \neq 0,\ \forall \mathcal{I}^n_{kij} \neq 0,\ n \geq 1. \end{aligned}$$

Such properties hold for $i, j, k \subset \mathbb{N}^+$.

Remark 2.5. Per 2.4 $i \wedge j \wedge k$ shall be non-zero values in $\mathbb{N}$. Moreover the identity shall hold for: $\mathcal{I}^n_{kij} \neq \mathcal{E}^n_{ijk}$.

Theorem 2.6. *There exist a sequence of products of manifold tensors $\mathcal{T}$ expressed as $\mathcal{B}$ indexed $i \in \mathbb{N}^+$:*

$$\mathcal{B}_i(n) = \prod_{k=1}^{n} \mathcal{T}^i_k + C_i,\ n \in \mathbb{N}^+. \tag{3}$$

Where $\mathcal{T}$ of index k is a tensor representing a manifold of four diemensional properties. And C is a residue constant in a arbitary n-plane. Moreover the inverse function of $\mathcal{B}_i(n)$ is:

$$\mathcal{B}_i(n)^{-1} = \pm \frac{\mathcal{I}^{-1}}{\prod_{k=1}^{n} \mathcal{T}^i_k + C_i},\ n, i \in \mathbb{N}^+. \tag{4}$$

Where $\mathcal{I}^{-1}$ is an non-zero inverse tensor.

3 Theorems, Definitions, and Several PDEs:

The following definition applies to Machine Learning for specific training techniques:

Definition 3.1. Consider the following function $\mathcal{G}$ being equal to:

$$\mathcal{G}(x) = \mathcal{T}\Big(\Big(\int_n^p Q(x)\,dx\Big) + \mathcal{D}_t\Big). \tag{5}$$

And the tensor $\mathcal{Z}$ being equal to the following Ricci tensor at $t \in \mathbb{N}^+$:

$$\mathcal{Z} = \frac{1}{4}\nabla \mathcal{R}_t. \tag{6}$$

Let following equation be in a linear 3-manifold space in $\mathbb{R}$:

$$\Delta \mathcal{X} = \sum_{n=1}^{\infty}\sum_{t=1}^{\infty} \mathcal{T}\Big(\Big(\int_n^p Q(x)\,dx\Big) + \mathcal{D}_t\Big) \pm \frac{1}{4}\nabla \mathcal{R}_t. \tag{7}$$

3.1 Several theorems per 3.1:

4 Conclusion:

Finishing of per previous definitions (2.3, 2.1), this note explores several identities and proofs regarding tensors.

References

[1] Pinter, C. C. *A book of abstract algebra.* Courier Corporation, 2010.

[2] Weil, A., and Weil, A. *Basic number theory*, vol. 1974. Springer, 1967.